International J. of Pure & Engg. Mathematics (IJPEM)
ISSN 2348-3881, Vol. 3 No. I (April, 2015), pp. 71-86

APPROXIMATE CONTROLLABILITY OF THE SEMILINEAR HEAT EQUATION WITH MEMORY AND DELAY

HUGO LEIVA[1] **AND NELSON MERENTES**[2]
[1] Universidad de los Andes
Facultad de Ciencias, Departamento de Matemática
Mérida 5101-Venezuela
[2] Universidad Central de Venezuela
Facultad de Ciencias, Departamento de Matemática
Caracas -Venezuela

Abstract

In this paper we apply Rothe's Fixed Point Theorem to prove the interior approximate controllability of the following semilinear Heat Equation with Memory and Delay

$$\begin{cases} \dfrac{\partial z(t,x)}{\partial t} = \Delta z(t,x) + 1_\omega u(t,x) & \\ + \int_0^t f(s, z(s-r,x), u(s,x))ds, & \text{in} \quad (0,\tau]\times\Omega, \\ z = 0, & \text{on} \quad (0,\tau)\times\partial\Omega, \\ z(s,x) = \phi(s,x), & s\in[-r,0], x\in\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$, $\phi : [-r, 0] \times \Omega \to \mathbb{R}$ is a continuous function, ω is an open nonempty subset of Ω, 1_ω denotes the characteristic function of the set ω, the distributed control u belongs to $L_2([0,\tau]; L_2(\Omega;))$ and the standard notation $z_t(x)$ defines a function from $[-r, 0]$ to $\mathbb{R}$ by $z_t(x)(s) = z(t + s, x), -r \leq s \leq 0$. Here $r \geq 0$ is the delay and the nonlinear function $f : [0,\tau] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is smooth enough and there are $\beta, b, c \in \mathbb{R}$ and $\frac{1}{2} \leq \beta < 1$ such that

$$|f(t,z,u)| \leq c|u|^\beta + b, \quad \forall u, z \in \mathbb{R}.$$

Under this condition we prove the following statement: For all open nonempty subset ω of Ω the system is approximately controllable on $[0,\tau]$. Moreover, we could exhibit a sequence of controls steering the nonlinear system from an initial state $\phi(0,\cdot)$ to an ϵ-neighborhood of the final state $z_1 \in L_2(\Omega)$ at time $\tau > 0$.

_____

Key Words : *Approximate controllability, Semilinear heat equation with memory and delay, Rothe's fixed point theorem.*

2010 AMS Subject Classification : Primary : 93B05, Secondary : 93C10.

© http: //www.ascent-journals.com